CSCI 5722 Computer Vision, Spring 2020

Student: Yinbo Chen

Instructor: Fleming

Homework 3

**Q1. True or False? Explain your reasoning. If the zero vector is part of a set of vectors, then that set is linearly dependent.**

It’s true. To assume that v is a vector space and consider **{0, v1, v2 , … , vn } vi ∈v.** To examine **c0\*0 + c1\*v1 + c2\*v2 +…+ cn\*vn = 0**. Let **c0 = 1** and **c1 = c2 = c3 … cn = 0**.So the equation can be expressed as **1\*0 + 0 \* v1 + 0 \*v2 + 0 \* v3 + … +0 \* vn = 0**. It shows that there exists **ci’**s not all zero and still satisfy the equation.

**Q2. True or False? Explain your reasoning. If a set of vectors is linearly dependent so is any larger set which contain it.**

It’s true. To assume that **p** is a vector space and consider**{p1,p2,p3,…,pn} pi ∈p** is dependent. And there are coefficients **a1, a2, … an,** are not all zero. The equation can be expressed as

**a1p1 + a2p2 + … + anpn= 0**

Now let **q** is **p’s** larger set and **m>= n**, and consider {**p1, p2, p3, …, pn** ,**pn+1, pn+2, … , pm**}. The equation of **q** can be expressed as

**b1p1 + b2p2 + … + bnpn+ bn+1pn+1 + bn+2pn+2 + … + bmpm = 0**

Take **b1 = a1, b2 = a2, … ,bn = an**, and **bn+1= bn+2 = … = bm = 0**. Because **ai’**s are not all zero, **bi**’s are not all zero either. We got the equations

**b1p1 + b2p2 + … + bmpm =a1p1 + a2p2 + … + anpn= 0**

So {**p1, p2, …, pm**} is linearly dependent as well.

**Q3. Show that convolving an image with a discrete, separable 2D filter kernel is equivalent to convolving with two 1D filter kernels. Estimate the number of operations saved for an N\*N image and a (2k + 1) \* (2k + 1) kernel.**

Let y[m,n] as the output image and x[m,n] is an input image. h[m,n] is a 2D filter. The 2D convolution is expressed below,

Y [m, n] = x [m, n] \* h [m, n]

The 2D filter can be decomposed into two 1D filters. The equation is:

h [m, n] = h1 [m] \* h2 [n]

So the final equation can be expressed as:

Y [m, n] = x [m, n] \* h1 [m] \* h2 [n]

As well as

Y [m, n] = x [m, n] \* h2 [n]\* h1 [m]

For example, in convolution a 2D filter with M\*N kernel. To assume if the kernel size is 3 \* 3 and it can be decomposed into (M \*1) and (1 \* N) matrices as below:

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So convolution with this separable kernel is equivalent to:

x[m, n] \* = x[m, n] \*

Number of operations saved:

Using a 2D filter convolves an N x N image with a (2k+1) x (2k+1) kernel. It totally needs N2 (2k+1)2 operations. But performing two 1D filters only needs N2(2(2k+1)). Let **i** equal to the saved operations, then:

i = N2 (2k+1)2 - N2(2(2k+1)) = N2(4k2 + 4k +1 -4K -2) =N2(4k2 - 1)

**Q4. What happens when we convolve a Gaussian with another Gaussian? Explain.**

We will get a wider Gaussian. The variance equals to the sum of two original variances. Consider that we have two Gaussians f1 and f2:

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**Q5. What is Dimensionality Reduction and how is it important with respect to Image Processing? Elaborate on at least one advantage and one disadvantage.**